The (noncommutative) geometry of difference equations

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Probably the two most important classes of special functions are the hypergeometric functions and (more recently) the Painlevé transcendents. Both of these are closely related to nice linear differential equations: the hypergeometric equations are determined by their singularities, while the Painlevé transcendents specify monodromy-preserving deformations of linear ODEs. This persists when considering discrete versions, replacing differential equations by difference, q-difference, or even elliptic difference equations. Thus to understand generalizations of hypergeometric functions or Painlevé, we need to understand moduli spaces of difference equations. I'll discuss an approach to this problem via an interpretation of difference equations as sheaves on noncommutative surfaces, and explain some useful consequences, including a classification of hypergeometric equations, and a new construction of Lax pairs for Painlevé equations.